Programme and abstracts booklet for talks at the
26th International Workshop on Matrices and Statistics (IWMS-2018)

edited & prepared by:

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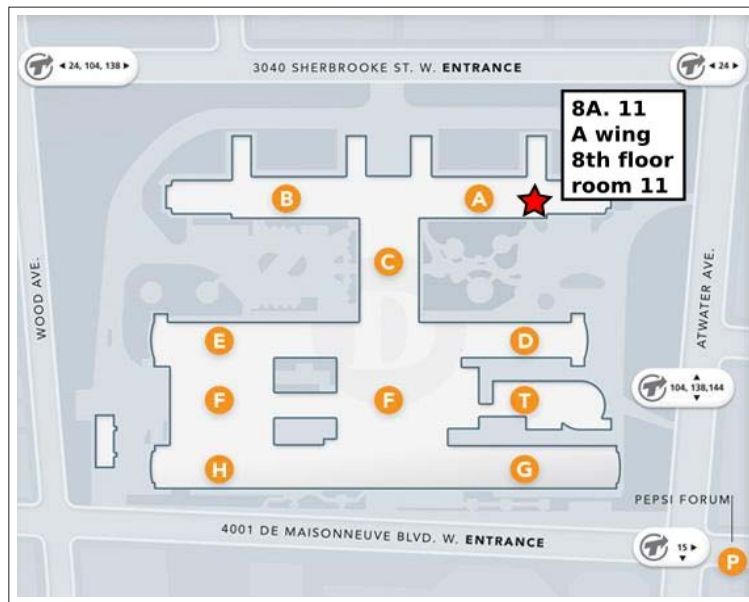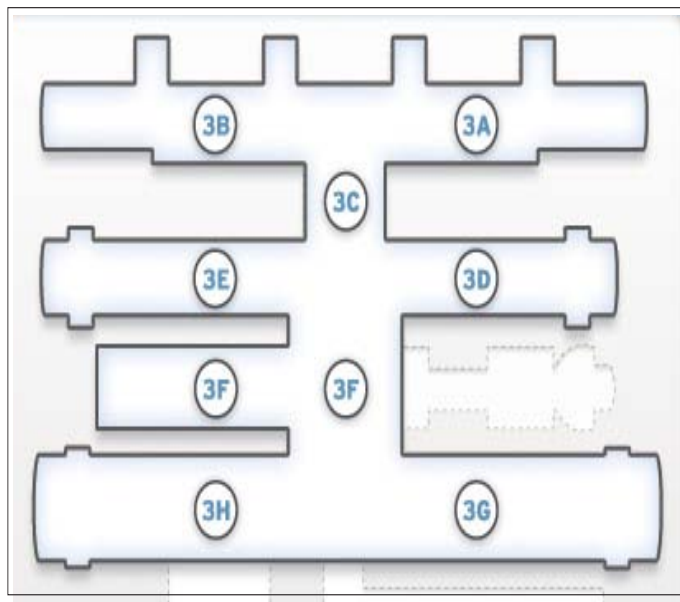& George P. H. Styan, McGill University, Montréal (Québec), Canada: `geostyan@gmail.com`

0.1. IWMS/5–7 June 2018: Introduction. The International Scientific and Organizing Committees of the 26th International Workshop on Matrices and Statistics (IWMS-2018) is delighted to have you join us at IWMS-2018 in Montréal (5–7 June 2018) to be held in the Multimedia Centre Rooms 3F.37, 3F.38 and 3F.43 (third floor) at Dawson College, 3040 rue Sherbrooke ouest, Westmount/Montréal.

On Tuesday 5 June 2018 we will hold a mini-symposium celebrating the 100th birth anniversary of Theodore Wilbur Anderson, Jr. (1918–2016). On Wednesday 6 June 2018 several talks by invited speakers will be presented, as well as some contributed talks. On Thursday 7 June 2018 we will hold the Third mini-symposium on "Magic squares, prime numbers and postage stamps" (IWMS-2018/M3). The first two mini-symposia were held in Madeira, Portugal (2016) and in Westmount/Montréal (2017). For more about the IWMS-2018 Workshop please visit our IWMS-2018 website `https://profchu.wixsite.com/mysite`

Our IWMS-2018 logo comprises $h = 92$ philatelic-friendly most-perfect pandiagonal 4×4 fully-magic matrices with equal magic sums $m_{92} = 2946$ and entries the $n_{92} = 1472$ consecutive integers from $1 - 1472$. Our logo follows up on the logo with $h = 53, m_{53} = 1698, n_{53} = 848$ entitled " $\star$Happy 2018$\star$ Equal Sums Magic Squares of Order 4", published by Inder J. Taneja, *Numbers Magic: Magic Squares, Selfie Numbers and Information Measures* online at `https://inderjtaneja.com/2017/12/29/2018-equal-sums-magic-squares-of-order-4/`

Contents

0.2. Multimedia Centre Rooms 3F.37 and 3F.38 (third floor) at Dawson College. All invited and contributed talks at IWMS-2018 in Westmount/Montréal (5–7 June 2018) will be held in the Multimedia Centre Rooms 3F.37 and 3F.38 (third floor) at Dawson College, 3040 rue Sherbrooke ouest, Westmount/Montréal. We have requested a special password for wifi internet access at Dawson College from 5–7 June 2018: username `guestmathws` with password `Dawson2018`

For parking at Dawson College use the south entrance at 4001 boulevard de Maisonneuve ouest (see right panel above: lower left between H and Wood Avenue). Other parking options include metered parking near the campus on the street. Indoor parking at the Alexis-Nihon shopping mall just opposite Dawson College; the entrance to Alexis-Nihon is on rue Ste-Catherine ouest. Or park at the Pepsi Forum de Montréal (see right panel: P lower right). The Métro station for Dawson and Alexis-Nihon is Atwater. The photo (below) of Dawson College is of the main north entrance at 3040 rue Sherbrooke ouest.

0.3. **International Scientific Committee.** S. Ejaz Ahmed (co-chair), Hans Joachim Werner (co-chair), George P. H. Styan (honorary chair), Francisco Carvalho, Katarzyna Filipiak, Jeffrey J. Hunter, Daniel Klein, Augustyn Markiewicz, Simo Puntanen, Dietrich von Rosen, Júlia Volaufová.

0.4. **International Organizing Committee.** Ka Lok Chu (chair), Simo Puntanen, George P. H. Styan.

0.5. **Participants.** Joanna Baksalary, Oskar Maria Baksalary, Adi Ben-Israel, Dennis S. Bernstein, Ka Lok Chu, S. W. Drury, J. P. David Dufour, Jorge Delgado Gracia, Alexander Hariton, Joshua Hum, Michelle Kuan, Yonghui Liu, Mika Mattila, Christopher C. Paige, Juan Manuel Peña, Simo Puntanen, Soile Puntanen, Evelyn Matheson Styan, George P. H. Styan, Yongge Tian, Hans Joachim Werner, Magdala Werner, Douglas P. Wiens, Marilyn Wiens.

0.6. **Acknowledgements.** We are most grateful to Karina D'Elmo and Myriam Dimanche for their help with local arrangements at Dawson College.

1. Timetable & Programme

1.1. **Tuesday, 5 June 2018 (11:00 am/11h00): Registration with pre-lunch coffee break in room 3F.43 and lunch break.**

1.2. **Tuesday, 5 June 2018 (2:00 pm/14h00): Mini-symposium celebrating the 100th birth anniversary of Theodore Wilbur Anderson, Jr. (1918–2016) on 5 June 1918. rooms 3F.37, 3F.38 & 3F.43.**
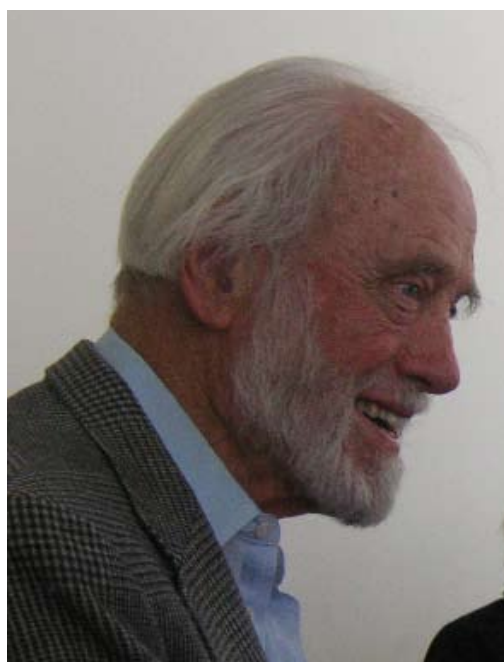
T. W. Anderson (June 2008)   *The first edition (1958)*

Tuesday, 5 June 2018 (2:00 pm/14h00): Mini-symposium celebrating the 100th birth anniversary of Theodore Wilbur Anderson, Jr. (1918–2016) on 5 June 1918. rooms 3F.37, 3F.38 & 3F.43.

Presentations will include (updated versions) of

- "Some comments on T. W. Anderson's publications", by George P. H. Styan, with updates for George Boole, Donald A. Darling & Milton Friedman:
- "An annotated and illustrated bibliography with hyperlinks for T. W. Anderson in celebration of his 90th birthday", by Simo Puntanen & George P. H. Styan,

based on presentations at the 17th International Workshop on Matrices and Statistics in Honour of Professor T. W. Anderson's 90th Birthday (IWMS-2008), Tomar, Portugal, 23–26 July 2008.

1.3. **"Theodore Anderson, 98; Paved Way for Modern Data Analysis", by Regina Nuzzo: The New York Times, vol. CLXVI, no. 57,381, page B5 (Monday, October 10, 2016).**

"All the News That's Fit to Print"

The New York Times

New England Edition
Today, sunshine and patchy clouds, breezy, cooler, high 58. **Tonight,** clear, chilly, low 44. **Tomorrow,** partly sunny, another cool day, high 60. Weather map is on Page A16.

VOL. CLXVI .... No. 57,381 + © 2016 The New York Times Company   MONDAY, OCTOBER 10, 2016 $2.50

THE NEW YORK TIMES, MONDAY, OCTOBER 10, 2016   Y   B5

Theodore Anderson, 98; Paved Way for Modern Data Analysis

By REGINA NUZZO

Theodore W. Anderson, a statistician whose work brought a new mathematical rigor to economics and social science in the postwar years and helped pave the way for modern econometrics and data analysis, died on Sept. 17 in Stanford, Calif. He was 98.

The cause was heart failure, his son, Robert, said.

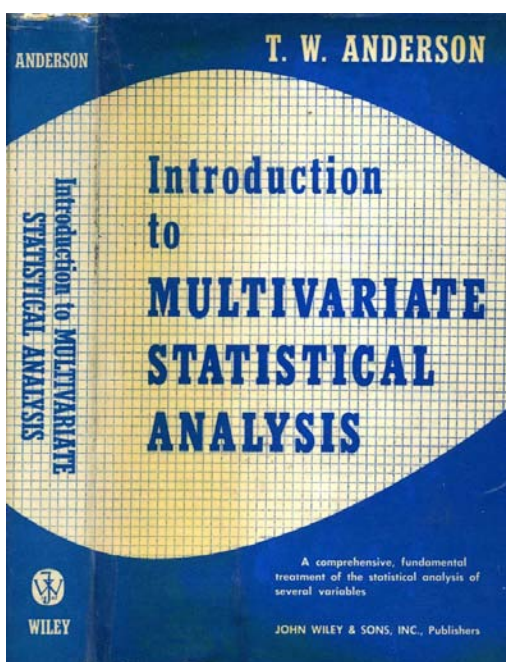Professor Anderson, the son of a Minnesota minister, had a distinguished academic career. He studied at Northwestern and Princeton, did research at the University of Chicago and taught at Columbia and, for more than two decades, at Stanford University.

During World War II, at Princeton, he did war research work on long-range weather forecasting, gunfire strategies for battleships and explosives testing.

But his signal work began immediately after the war, when he joined pioneering efforts at the University of Chicago to help shape postwar economic policy decisions.

The work involved developing systems of mathematical equations that would reveal underlying structures in the economy. Those systems evolved into econometric models widely used today by the Federal Reserve and other central banks.

Professor Anderson also made advances in the analysis of data in psychology and the social sciences.

His talent for reducing complicated systems to their mathematical essence helped him strengthen tools for dealing with the sorts of large data sets common in the social sciences, in which researchers look for patterns within a broad swath of information collected on individuals.

In one instance, in 1961, psychologists applied these methods to data derived from detailed personality questionnaires and concluded that individual personalities could be summed up by an overarching "big five" traits .

Professor Anderson's contributions lay the groundwork for an advanced analysis of data sets containing not just dozens but thousands of variables.

"Among other things, I would say that Ted was a prophet of the big data era," David Donoho, a professor of statistics at Stanford and a longtime colleague, told the Stanford University news office.

Professor Anderson's book "An Introduction to Multivariate Statistical Analysis" (1958) remains a classic in the field, educating generations of statisticians in the conceptual underpinnings of a particularly challenging kind of data analysis.

In multivariate data, many variables are considered simultaneously rather than one at a time. Using this type of analysis in medicine, a person's health is gauged not just by blood pressure, for example, but by blood pressure in tandem with weight, cholesterol levels and heart rate.

Translated into Russian in 1964 and appearing in a third edition in 2003, the book has inspired thousands of technical papers. Michael Perlman, a professor of statistics at the University of

Washington, said that the book is remarkable in that it can lead a reader with very little mathematical background through a clear and logical progression of the entire field.

"I believe that science progresses more if the communication is made easier," Professor Anderson told an interviewer in 1986. "It's unfair to the reader, as well as the editor, to put out papers which are difficult to read, not because of the difficulty of the material but

A mathematician who developed equations that revealed patterns in the economy.

because of the sloppiness of the work and the carelessness in exposition."

He also brought mathematical rigor to the study of sets of measurements taken over time — of unemployment numbers, for example, which depend strongly on the previous year's numbers, which in turn depend on those of the year before. In the 1960s, economists were struggling with these mathematically complex "time series," which were vital for capturing the dynamic nature of the economy.

Professor Anderson's work, including his book "The Statistical Analysis of Time Series" (1971), was at the forefront of this new field, John Taylor, an economics

professor at Stanford and a former doctoral student of Professor Anderson's, said in an interview.

Time series analyses are now "very commonplace in economics," he said, "and Ted's work helped move that forward."

Professor Anderson's early interest in economics and social science applications stemmed from his goal of "doing some good," he said in the 1986 interview.

"While I find intellectual interest in mathematical statistical problems," he said, "I think a final objective is to have an effect on analyzing data and decision making under uncertainty."

Theodore Wilbur Anderson was born in Minneapolis on June 5, 1918, the oldest of three children of Theodore Anderson Sr., a minister and educator, and the former Evelynn Johnson. He was inspired by a high school teacher to study chemistry, first at North Park College in Chicago and then at Northwestern University in Evanston, Ill., until he realized he was, as he later said, "terrible in the laboratory."

He switched to mathematics and took all his required math courses in one year, graduating in 1939. He then worked with Samuel Wilks, a leading mathematical statistician, at Princeton, where he obtained both a master's and a Ph.D. in mathematics.

At Princeton he was exposed to the application of mathematics and statistics to other fields, including the military. His war research work on weather forecasting and armaments was done under the auspices of the Applied

Mathematics Panel of the National Defense Research Committee.

At Princeton he came under the influence of leading statisticians like William Cochran, Will Dixon, John Tukey and Charles Winsor and took mathematical economics seminars with Oskar Morgenstern, who was at the time writing a book with John van Neumann on game theory and economic behavior.

After leaving Princeton in 1945 he joined the Cowles Commission for Research in Economics at the University of Chicago, which was suffused, he said, by an "almost missionary zeal" to develop new approaches to economics through probability, statistics and mathematics. He worked with top scholars, many of whom would later receive the Nobel in economic science, among them Lawrence Klein, Leo Hurwicz and Tjalling Koopmans.

At Cowles he worked on tricky systems of economic equations that attempted to disentangle cause and effect by considering several related factors simultaneously.

Cowles researchers used these systems, for example, to analyze the postwar agricultural economy by looking at consumers' demand for food, household incomes and available food supply from farmers.

After a year in Chicago, the mathematician Abraham Wald enticed Professor Anderson to join the faculty in the new department of mathematical statistics at Columbia. He remained there un-

SIMO PUNTANEN

Theodore W. Anderson of Stanford University in 1988.

til 1967, when he moved with his family to Stanford for a joint appointment in statistics and economics.

Besides his son, Professor Anderson's survivors include his wife, the former Dorothy Fisher, whom he met at the University of Chicago; two daughters, Janet Anderson Yang and Jeanne Anderson; and five grandchildren.

Although he retired from classroom duties in 1988, Professor Anderson continued to give talks, attend seminars and do research from his home office. He recently submitted a technical paper and, just days before his death, was responding to peer reviewers' comments.

His son recalled him saying, "Well, they're wrong, but that is a good idea for a follow-up paper."

More obituaries appear on the following page.

1.4. Wednesday, 6 June 2018: Timetable for IWMS-2018 Invited & Contributed Talks [all talks to be held in the Multimedia Centre Rooms 3F.37 and/or 3F.38 (third floor) at Dawson College, 3040 rue Sherbrooke ouest, Westmount/Montréal].

- 09:30 am/09h30: S. Ejaz Ahmed: Post selection, estimation and prediction in sparse regression models (abstract 3.1)

- 10:00 am/10h00: Oskar Maria Baksalary & Götz Trenkler: Representations of the Moore–Penrose inverses of matrices (abstract 3.2)

- 10:30 am/10h30: Adi Ben-Israel: Matrix volume and its applications (abstract 3.3)

- 11:00 am/11h00: *Coffee break:* room 3F.43

- 11:30 am/11h30: Dennis S. Bernstein: Input estimation for linear discrete-time dynamical systems (abstract 3.4)

- 12:00 noon/12h00: S. W. Drury: Some old conjectures of Bapat and Sunder (abstract 3.5)

- 12:30 noon/12h30: Yonghui Liu: Estimation and influence diagnostics for an autoregressive model under skew-normal distributions (abstract 3.6)

- 1:00 pm/13h00: *Lunch break*

- 2:30 pm/14h30: Mika Mattila: The arithmetic Jacobian matrix and determinant (abstract 3.7)

- 3:00 pm/15h00: Christopher C. Paige: The effects of loss of orthogonality on large sparse matrix computations (abstract 3.8)

- 3:30 pm/15h30: *Coffee break:* room 3F.43

- 4:00 pm/16h00: Yongge Tian: Multilevel statistical models, least-squares estimators, and reverse-order laws for generalized inverses (abstract 3.9)

Wednesday, 6 June 2018: Timetable for IWMS-2018 Contributed Talks

- 4:30 pm/16h30: Jorge Delgado Gracia, Guillermo Peña & Juan Manuel Peña: Schoenmakers–Coffey matrices (abstract 4.1)

- 5:00 pm/17h00: Simo Puntanen & Augustyn Markiewicz: Linear prediction sufficiency in the misspecified linear model (abstract 4.2)

2. Third mini-symposium on magic squares, prime numbers & postage stamps
organized by Ka Lok Chu, Simo Puntanen & George P. H. Styan

2.1. Philatelic magic square for Götz Trenkler in celebration of his 75th birthday: 14 July 2018.
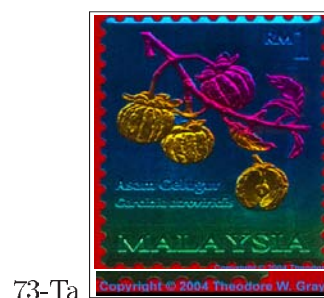

89-Ac   37-Rb   41-Nb   73-Ta

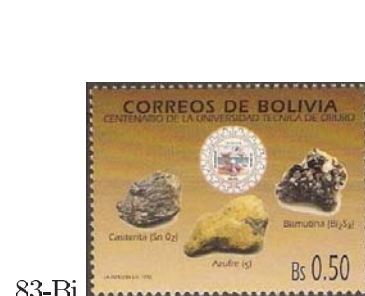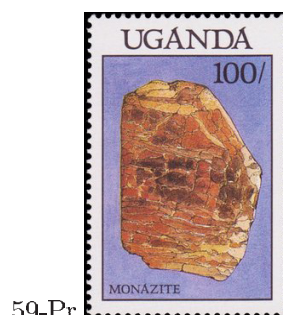61-Pm   53-I   109-Mt   17-Cl

79-Au   47-Ag   31-Ga   83-Bi

11-Na   103-Lr   59-Pr   67-Ho

2.2. **Thursday, 7 June 2018: Timetable for three talks in the IWMS-2018/M3 Mini-Symposium on "Magic squares, prime numbers and postage stamps".**

Three talks M3a, M3b, M3c by Ka Lok Chu, Simo Puntanen & George P. H. Styan:

- 10:00 am/10h00: Talk M3a: Some comments on a philatelic magic square for the Trenkler–Trenkler MPPD prime-magic matrix (abstract 6.3)

- 10:30 am/10h30: Talk M3b: Some comments on two logos and on some related curiosity- and philatelic-friendly numbers (abstract 6.4)

- 11:00 am/11h00: Talk M3c: Some comments on selected new mathematical postage stamps from USA, Italy and Croatia (abstract 6.5)

2.3. **Poster presentation by Richard William Farebrother.**

Poster P1: Symmetric and nonsymmetric square matrices with common row and column totals expressed as weighted sums of permutation matrices, illustrated philatelically (abstract 5.1)

2.4. **Poster presentations by Ka Lok Chu, Simo Puntanen & George P. H. Styan.**

Poster Q1: A philatelic magic square (with details) for Götz Trenkler celebrating his 75th birthday: 14 July 2018 (abstract 6.4 for Poster Q1 and talk M3a)

Poster Q2: Some comments on philatelic stuffed cheese (*keshi yena*): bon appétit! (abstract 6.5 for Poster Q2 and talk M3b)

Poster Q3: Some more comments on Wells's five most beautiful theorems (abstract 6.6 for Poster Q3 and talk M3c)

Poster Q4: A philatelic introduction to Dawson College, McGill University & Lord Strathcona (abstract 6.1)

Poster Q5: A philatelic introduction to Mersenne primes and perfect numbers (abstract 6.2)

Poster Q6: Magic square-friendly 19 (The final report). (abstract 6.3)

2.5. **Thursday, 7 June 2018: After-lunch excursion possibility — Dawson College Garden of Peace.**

3. Abstracts for IWMS-2018 invited talks

3.1. **S. Ejaz Ahmed: Post selection, estimation and prediction in sparse regression models.** Nowadays a huge amount of data is available, and the need for novel statistical strategies to analyze such data sets is pressing. This talk focuses on the development of statistical and computational strategies for a sparse regression model in the presence of mixed signals. The search for such signals, sometimes called networks or pathways, is for instance an important topic for those working on personalized medicine. We propose a new and unique "post selection shrinkage estimation strategy" that takes into account the joint impact of both strong and weak signals to improve the prediction accuracy, and opens pathways for further research in such scenarios. *E-mail address:* `sahmed5@brocku.ca`

3.2. **Oskar Maria Baksalary & Götz Trenkler: Representations of the Moore–Penrose inverses of matrices.** During the talk we will recall various results concerned with different representations of the Moore–Penrose inverses of matrices. We will look at the representations from different perspectives demonstrating diversity of possible approaches to the subject. Among the topics discussed will be the Moore–Penrose inverses of: matrices modified by matrices of rank-one, partitioned matrices, functions of a pair of Hermitian idempotent matrices, and functions of a square matrix represented by the Hartwig–Spindelböck decomposition. Furthermore, some of the results will shed light on the links between the Moore–Penrose inverse and other generalized inverses, such as Bott–Duffin or core inverses. A number of examples dealing with applications of the Moore–Penrose inverse in various areas of research (e.g., physics) will be discussed as well. *E-mail address:* `obaksalary@gmail.com`

3.3. **Adi Ben-Israel: Matrix volume and its applications.** *E-mail address:* `adi.benisrael@gmail.com`

The **volume** vol(A) of a matrix $A \in \mathbb{R}_r^{m \times n}$ is, [1]:

(a) the product of the r singular values of A, or equivalently,
(b) the square root of the sum of squares of all $r \times r$ subdeterminants of A, or equivalently,
(c) the volume of the image under A of a unit cube in the $R(A^T)$.

The matrix volume is a generalization to rectangular matrices of the absolute value of the determinant. Definition (b) is applicable to non-numerical matrices, in particular to singular or rectangular Jacobians. The following topics and representative applications will be discussed (time permitting):

- Angles and projections
- Multilinear algebra
- Compound matrices, and how to compute them with Maple
- The "ultimate" SVD
- Differential geometry of surfaces: Gauss' 1st fundamental form
- Change-of-variables in integration [2]
- Concentration of measure: The geometry of the sphere in $\mathbb{R}^n$, $n >> 1$.
- Applications to probability [3], [6]
- Low rank approximation of matrices via volume sampling [4]
- Face recognition [5], [7]

References

[1] A. B-I, A volume associated with $m \times n$ matrices, *Lin. Algeb. Appl.* **167**(1992), 87–111 [PDF]
[2] A. B-I, The change of variables formula using matrix volume, *SIAM Journal on Matrix Analysis* **21**(1999), 300–312 [PDF]
[3] A. B-I, An application of the matrix volume in probability, *Lin. Algeb. Appl.* **321**(2001), 9–25 [PDF]
[4] A. Deshpande, L. Rademacher et al, Matrix approximation and projective clustering via volume sampling, *Theory of Computing* **2**(2006), 225–247
[5] J. Meng and W. Zhang, Volume measure in 2DPCA–based face recognition, *Pattern Recognition* **28**(2007), 1203–1208
[6] I. Weissman, Sum of squares of uniform random variables, *Statistics and Probability Letters* (To appear)
[7] Z. Xu, J. Zhang and X. Dai, Boosting for learning a similarity measure in 2DPCA based face recognition, *2009 World Congress on Computer Science and Information Engineering*, IEEE

Note added in proof: [6] I. Weissman: Sum of squares of uniform random variables, *Statistics & Probability Letters,* vol. 129, pp. 147–154 (October 2017).

3.4. **Dennis S. Bernstein: Input estimation for linear discrete-time dynamical systems.** Given the output of a system, can we find the input? For static maps, we need only construct a left inverse. This talk will focus on challenges that arise when the system has dynamics. These challenges include the stability and causality of the left inverse, knowledge of the initial condition, the achievable estimation delay, and the need to avoid unstable pole-zero cancellation. It will be shown that, for linear discrete-time systems, the answers to these questions depend on the analysis of a block-Toeplitz matrix. Finite-time and asymptotic input estimation will be shown for various classes of systems. *E-mail address:* `dsbaero@umich.edu`

3.5. **S. W. Drury: Some old conjectures of Bapat and Sunder.** In the mid 1980s, Bapat and Sunder made two conjectures concerning the permanents of positive semidefinite matrices. Recently both of these conjectures have been shown to be false. We will present the counterexamples, place them in context and discuss some of the open proble ms in this area. *E-mail address:* `<swdrury@hotmail.ca>`

3.6. **Yonghui Liu: Estimation and influence diagnostics for an autoregressive model under skew-normal distributions.** Autoregressive models have played an important role in time series. In this paper, an autoregressive model under skew-normal distributions is considered. The estimation of parameters in the model is studied based on the EM algorithm and the local influence method is used to conduct statistical diagnostics. The normal curvature diagnostics for the model under four perturbation schemes for identifying possible influential observations are established. After a simulation study is conducted and the performance of the proposed method is evaluated, an example of real-data analysis is presented and discussed. Our model is used to analyse weekly log-return data for Chevron shares with our local influence analysis conducted to improve the model fit. *E-mail address:* `liuyh@lsec.cc.ac.cn`

3.7. **Mika Mattila: The arithmetic Jacobian matrix and determinant.** Let $a_1, \ldots, a_m$ be such real numbers that can be expressed as a finite product of prime powers with rational exponents. Using arithmetic partial derivatives, we define the arithmetic Jacobian matrix $\mathbf{J_a}$ of the vector $\mathbf{a} = (a_1, \ldots, a_m)$ analogously to the Jacobian matrix $\mathbf{J_f}$ of a vector function $\mathbf{f}$. We introduce the concept of multiplicative independence of $\{a_1, \ldots, a_m\}$ and show that $\mathbf{J_a}$ plays in it a similar role as $\mathbf{J_f}$ does in functional independence. We also present a kind of arithmetic implicit function theorem and show that $\mathbf{J_a}$ applies to it somewhat analogously as $\mathbf{J_f}$ applies to the ordinary implicit function theorem. *E-mail address:* `mika.mattila@tut.fi`

3.8. **Christopher C. Paige: The effects of loss of orthogonality on large sparse matrix computations.** Many useful large sparse matrix algorithms are based on orthogonality, but for efficiency this orthogonality is often obtained via short term recurrences. This can lead to both loss of orthogonality and loss of linear independence of computed vectors, yet with well designed algorithms high accuracy can still be obtained. Here we discuss a nice theoretical indicator of loss of orthogonality and linear independence and show how it leads to a related higher dimensional orthogonality that can be used to analyze and prove the effectiveness of such algorithms. We illustrate advantages and shortcomings of such algorithms with Cornelius Lanczos' symmetric matrix tridiagonalization process. This is the basis for several very useful large sparse matrix algorithms, and is itself very effective for solving the eigenproblem and solution of equations problems for large sparse symmetric matrices. The talk is expository, summarizing other work, and avoiding detailed error analyses. *Key words & phrases:* orthogonality, large sparse matrices, Lanczos process. *E-mail address:* `chris@cs.math.mcgill`

3.9. **Yongge Tian: Multilevel statistical models, least-squares estimators, and reverse-order laws for generalized inverses.** A linear statistical model may involve unknown parameters that vary at more than one level. In such a case, multilevel statistical modeling is an analytical technique for estimating regression and related models with data that have a hierarchical structure. In this talk, I first introduce two kinds of ordinary least-squares estimators under multilevel linear models, and describe the equivalence of these estimators by using various matrix identities composed by generalized inverses. I then show a family of equivalent statements for the matrix identities to hold by using matrix rank optimization method. The whole work demonstrates that many facts and results in mathematics have a certain essential interpretation in statistics and can be used to solve many fundamental problems in statistical inference. *E-mail address:* `yongge.tian@gmail.com`

4. Abstracts for IWMS-2018 contributed talks

4.1. **Jorge Delgado Gracia, Guillermo Peña & Juan Manuel Peña: Schoenmakers–Coffey matrices.** We will discuss Schoenmakers–Coffey matrices which are correlation matrices with important financial applications. We present several characterizations of positive extended Schoenmakers–Coffey matrices. Several numerical methods for algebraic computations with Schoenmakers–Coffey matrices are analyzed and compared.

Key words & phrases: Algebraic computations, correlation matrices, financial applications, Schoenmakers–Coffey matrices. *E-mail address:* `jorgedel@unizar.es`

4.2. **Simo Puntanen & Augustyn Markiewicz: Linear prediction sufficiency in the misspecified linear model.** We consider the general linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, or shortly $\mathcal{M} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$, supplemented with the future unobservable random vector $\mathbf{y}_*$, coming from $\mathbf{y}_* = \mathbf{X}_*\boldsymbol{\beta} + \boldsymbol{\varepsilon}_*$, where the expectation of $\mathbf{y}_*$ is $\mathbf{X}_*\boldsymbol{\beta}$ and the covariance matrix of $\mathbf{y}_*$ is known as well as the cross-covariance matrix between $\mathbf{y}_*$ and $\mathbf{y}$. We denote the supplemented model as $\mathcal{M}_*$. The misspecified supplemented model is denoted as $\underline{\mathcal{M}}$, and the misspecification concerns the covariance part of the setup. Suppose that $\mathbf{Fy}$ is linearly sufficient for estimable parametric function $\mathbf{X}_*\boldsymbol{\beta}$ under $\mathcal{M}$. We give necessary and sufficient conditions that $\mathbf{Fy}$ continues to be linearly sufficient for $\mathbf{X}_*\boldsymbol{\beta}$ under the model $\underline{\mathcal{M}}$. The corresponding properties regarding the linear sufficiency with respect to $\boldsymbol{\varepsilon}_*$ and $\mathbf{y}_*$ are also studied.

Key words & phrases: Linear models, regression designs, continuous designs, exact designs, information matrix, equivalence, domination. *E-mail address:* `simo.puntanen@uta.fi`

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5. Abstracts for the 3rd international mini-symposium on "Magic squares, prime numbers & postage stamps" (IWMS-2018/M3)

5.1. **Richard William Farebrother: Symmetric and nonsymmetric square matrices with common row and column totals expressed as weighted sums of permutation matrices, illustrated philatelically: Poster P1.** This article is concerned with the possibility of writing an $m \times m$ matrix whose rows and columns each sum to a common value as a weighted sum of $m \times m$ permutation matrices. Although our examples are restricted to the 3×3 case, the technique is entirely general and readers are invited to apply it to magic and non-magic square matrices of higher orders. *Key words and phrases:* Lewis Carroll (1832–1898), linear combination, magic square, permutation matrix. *E-mail address:* R.W.Farebrother@Hotmail.com

References:

[1] Richard William Farebrother (2015), "Supplementary Notes on Lewis Carroll, Graeco-Latin Squares and Magic Squares with an Annexe on Maria Theresa Thalers and British Banknotes", *Acta et Commentationes Universitatis Tartuensis de Mathematica,* vol 19, no. 2, pp. 97–107.

[2] R. William Farebrother, Shane T. Jensen & George P. H. Styan (2000). "Charles Lutwidge Dodgson: a biographical and philatelic note", *Image: The Bulletin of the Internationsl Linear Algebra Society*, no. 25 (October 2000), pp. 22–23.

[3] Edward Wakeling (1992), *Lewis Carroll's Games and Puzzles,* Dover Publications, New York, in association with the Lewis Carroll Birthplace Trust, Daresbury, Cheshire, England.

6. Presentations by Ka Lok Chu, Simo Puntanen & George P. H. Styan

6.1. **Poster Q4: A philatelic introduction to Dawson College, McGill University and Lord Strathcona (1820–1914).** We have found no philatelic items for Dawson College or for McGill University *per se* but we have found postage stamps for Dawson City (Yukon), McGill Cab Stand (Kathleen Morris), and for Macdonald College (Macdonald Campus of McGill University). Dawson College is an English-language CEGEP (*Collège d'enseignement général et professionnel*) located in Westmount/Montréal (Québec), Canada. It was named after Sir John William Dawson (1820–1899) who was principal of McGill University (1855–1893). Dawson City in the Yukon Territory, Canada, is inseparably linked to the Klondike Gold Rush (1896–1899). The current settlement was named in January 1897 after the Canadian geologist and surveyor George Mercer Dawson (1849–1901), eldest son of Sir John William Dawson. We conjecture that the "McGill Cab Stand" depicted in the Christmas-card painting (1927) by Kathleen Moir Morris (1893–1986) on the stamp (centre panel) may have been located on the south side of Sherbrooke Street West near Strathcona Hall and opposite the Roddick Gates entrance to McGill University [1[.
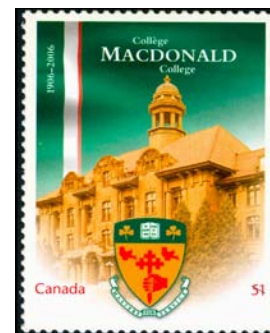
Reference: [1] *Discover Montreal: An Architectural and Historical Guide,* by Joshua Wolfe & Cécile Grenier, Newly Revised 3rd Edition, Éditions Libre Expression (1991). [First edition (1983), 2nd edition (1987).]

6.2. Poster Q5: A philatelic introduction to Mersenne primes and perfect numbers.

We celebrate the discovery [1] by Jonathan Pace (Germantown, Tennessee, USA) on Boxing Day 26 December 2017 of the Mersenne prime $2^p - 1$ with the prime $p = 277232917$. This prime number is the largest prime number known as of June 15, 2018 and the 50th largest Mersenne prime and was found with the Great Internet Mersenne Prime Search (GIMPS) project. Founded in 1996, GIMPS has discovered the last 16 Mersenne primes.

The even number π_p is a perfect number if and only if $\pi_p = 2^{p-1}(2^p - 1)$ with p and $2^p - 1$ prime and so with the discovery of this new Mersenne prime we see that the even number π_p with $p = 277232917$ is perfect!

Key words and phrases: 50th Mersenne prime, Great Internet Mersenne Prime Search (GIMPS) project, Jonathan "Jon" Pace, meter postmark, perfect numbers, *Prime Curios!* [2], prime numbers, Sophie Germain prime, Urbana, Illinois.

Reference:
[1] "A 14-year search ends in church: a church deacon found the biggest prime number yet, and it wasn't as hard as you'd think", by Valencia Prashad, *The New York Times,* New England print edition, page D3 (30 January 2018).

6.3. Poster Q6: Magic square-friendly 19. The final report.

In his *Prime Curios:* 19 online[1] Chris Caldwell noted that "The recurring decimal cycles for $\frac{1}{19}$ to $\frac{18}{19}$ form a true magic square," which Wells (2005, p. 39)[2] gave explicitly.

We find that the $2n \times 2n$ matrix $\mathbf{M}_{(2n)}$

$$\mathbf{M}_{(2n)} = \begin{pmatrix} \mathbf{M}_{11} & \frac{m}{n}\mathbf{E} - \mathbf{M}_{11} \\ \frac{m}{n}\mathbf{E} - \mathbf{F}\mathbf{M}_{11} & \mathbf{F}\mathbf{M}_{11} \end{pmatrix} \qquad (6.1)$$

is fully-magic with magic sum $m = \operatorname{tr}(\mathbf{I} + \mathbf{F})\mathbf{M}_{11}$ for any top-left $n \times n$ submatrix $\mathbf{M}_{11}$. Here $\mathbf{F}$ is the flip matrix and every entry of $\mathbf{E}$ is equal to 1.

The 18×18 "Caldwell–Wells recurring decimal cycles magic square" is a special case of $\mathbf{M}_{(2n)}$ (6.1) with $n = 9$ and $m = 81$ and

$$\mathbf{M}_{11} = \begin{pmatrix} 0 & 5 & 2 & 6 & 3 & 1 & 5 & 7 & 8 \\ 1 & 0 & 5 & 2 & 6 & 3 & 1 & 5 & 7 \\ 1 & 5 & 7 & 8 & 9 & 4 & 7 & 3 & 6 \\ 2 & 1 & 0 & 5 & 2 & 6 & 3 & 1 & 5 \\ 2 & 6 & 3 & 1 & 5 & 7 & 8 & 9 & 4 \\ 3 & 1 & 5 & 7 & 8 & 9 & 4 & 7 & 3 \\ 3 & 6 & 8 & 4 & 2 & 1 & 0 & 5 & 2 \\ 4 & 2 & 1 & 0 & 5 & 2 & 6 & 3 & 1 \\ 4 & 7 & 3 & 6 & 8 & 4 & 2 & 1 & 0 \end{pmatrix}.$$

When $\mathbf{M}_{11}$ is nonsingular then the Schur complement $(\mathbf{M}_{(2n)}/\mathbf{M}_{11}) = \frac{m}{n}(2 - \frac{m}{n}\mathbf{e}'\mathbf{M}_{11}^{-1}\mathbf{e})\mathbf{E}$. When $\mathbf{M}_{11}$ is fully-magic with magic sum $m_1 = \frac{1}{2}m$ then the Schur complement $(\mathbf{M}_{(2n)}/\mathbf{M}_{11}) = \mathbf{0}$ and $\mathbf{M}_{(2n)}$ has rank n.

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[1] http://primes.utm.edu/ caldwell/curios/page.php?rank=56
[2] *Prime Numbers: The Most Mysterious Figures in Math,* by David Wells (2005).

6.4. **Talk M3a & Poster Q1: A philatelic magic square for the chemical-friendly 4×4 Trenkler–Trenkler most-perfect pandiagonal prime-magic square.**

All 16 entries of the magic matrix

$$\mathbf{M}_{\mathrm{TT}} = \begin{pmatrix} 89 & 37 & 41 & 73 \\ 61 & 53 & 109 & 17 \\ 79 & 47 & 31 & 83 \\ 11 & 103 & 59 & 67 \end{pmatrix} \qquad (6.2)$$

created by the brothers Dietrich and Götz Trenkler (c. 2002) are prime numbers and all are chemical-friendly:

- Na 11 Sodium
- Cl 17 Chlorine
- Ga 31 Gallium
- Rb 37 Rubidium
- Nb 41 Niobium
- Ag 47 Silver
- I 53 Iodine
- Pr 59 Praseodymium
- Pm 61 Promethium
- Ho 67 Holmium
- Ta 73 Tantalum
- Au 79 Gold
- Bi 83 Bismuth
- Ac 89 Actinium
- Lr 103 Lawrencium
- Mt 109 Meitnerium

This enabled us to construct the Trenkler–Trenkler most-perfect pandiagonal prime-magic square defined by (6.2) above. The chemical-friendly prime-magic matrix $\mathbf{M}_{\mathrm{TT}}$ uses the 16 prime-number TT-set

11, 17, 31, 37, 41, 47, 53, 59, 61, 67, 73, 79, 83, 89, 103, 109.

Charles D. Shuldham [2 (1914)] reported that on Christmas Day 25 December 1911, Lorraine Screven Frierson (1861–1936) created a 4×4 prime-magic square which we define by the most-perfect pandiagonal magic matrix

$$\mathbf{M}_{\mathrm{FS}} = \begin{pmatrix} 73 & 41 & 13 & 113 \\ 23 & 103 & 83 & 31 \\ 107 & 7 & 47 & 79 \\ 37 & 89 & 97 & 17 \end{pmatrix}. \qquad (6.3)$$

The Frierson–Shuldham matrix $\mathbf{M}_{\mathrm{FS}}$ is chemical-friendly and uses the 16 prime-number FS-set

7, 13, 17, 23, 31, 37, 41, 47, 73, 79, 83, 89, 97, 103, 107, 113.

This FS-set is "wider" than the TT-set (given sbove). However, both $\mathbf{M}_{\mathrm{TT}}$ and $\mathbf{M}_{\mathrm{FS}}$ have magic sum 240 and both are keyed with rank 3, index 1.

We assemble the 16 maximal atomic numbers $103, 104, \ldots, 118$ [3 (2018)] as a 4×4 philatelic magic square according to the 4×4 EP most-perfect pandiagonal matrix $\mathbf{A}$ (6.4) with maximal magic sum 442

$$\mathbf{A} = \begin{pmatrix} 103 & 110 & 115 & 114 \\ 116 & 113 & 104 & 109 \\ 106 & 107 & 118 & 111 \\ 117 & 112 & 105 & 108 \end{pmatrix}. \qquad (6.4)$$

References:
[1] Geoffrey Boocock's "Where chemistry meets maths meets art", "Community News", *Voice: The Quarterly Magazine of the Royal Society of Chemistry,* Issue 01 (10 July 2017), original online version: http: //www.rsc.org/news-events/community/ 2017/jul/chemistry-maths-art/, shorter print version: page 21 (October 2017).

[2] "Associated prime number magic squares", by Charles D. Shuldham, *The Monist: A Quarterly Magazine Devoted to the Philosophy of Science,* vol. 24, no. 3, pp. 472–475 (July 1914).

[3] "Row 7 of the periodic table complete: can we expect more new elements; and if so, when?", by Jan Reedijk [*Polyhedron,* vol. 141, pp. 1–4 (2018)

6.5. Talk M3b & Poster Q2: Two logos and some related curiosity- and philatelic-friendly numbers. Our main result is the formula (6.5) for the "Taneja set" of h most-perfect pandiagonal 4×4 fully-magic matrices with equal magic sums $m_h = 2(16h+1)$ and its $16h$ entries the $n_h = 16h$ consecutive integers from 1 through $n_h = 16h$.

$$\mathbf{T}_j^{(h)} = h \begin{pmatrix} 1 & 8 & 13 & 12 \\ 14 & 11 & 2 & 7 \\ 4 & 5 & 16 & 9 \\ 15 & 10 & 3 & 6 \end{pmatrix} - (h-j) \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

$$- (j-1) \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}; \qquad j = 1, 2, \ldots, h. \tag{6.5}$$

We may write (6.5) as

$$\mathbf{T}_j^{(h)} = h\mathbf{M} - (h-j)\mathbf{B} - (j-1)(\mathbf{E} - \mathbf{B})$$

$$= h\mathbf{M} - (h + 1 - 2j)\mathbf{B} - (j-1)\mathbf{E} \tag{6.6}$$

where $j = 1, 2, \ldots, h$ and

$$\mathbf{M} = \begin{pmatrix} 1 & 8 & 13 & 12 \\ 14 & 11 & 2 & 7 \\ 4 & 5 & 16 & 9 \\ 15 & 10 & 3 & 6 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}. \tag{6.7}$$

We will call $\mathbf{M}$ the "parent matrix" for $\mathbf{T}_j^{(h)}$ (6.5) and $\mathbf{B}$ the "odd/even binary partner" for $\mathbf{M}$. The matrix $\mathbf{E}$ has every entry equal to 1.

With $h = 53$ in (6.5), we obtain the set of matrices for Taneja's Happy 2018 logo (6.8) left panel with $n_h = n_{53} = 16h = 16 \times 53 = 848$, $m_h = m_{53} = 2(16h + 1) = 2 \times 849 = 1698$ as in (6.8) left panel and reference [1]. Here each of the integers from 1–848, with $848 = 16 \times 53$, is represented precisely once.
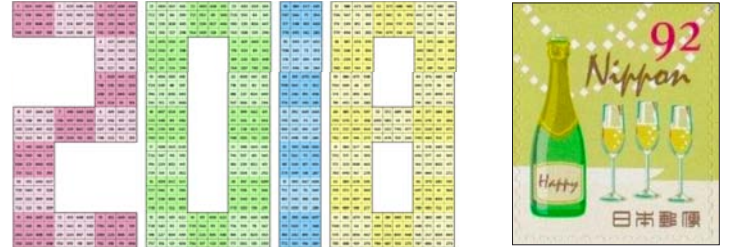
(6.8)

With $h = 92$ in (6.5), however, we obtain the set of matrices for our IWMS-2018 logo (6.9) below with $n_h = n_{92} = 16h = 16 \times 92 = 1472$, $m_h = m_{92} = 2(16h + 1) = 2 \times 1473 = 2946$. Here each of the integers from 1–1472, with $1472 = 16 \times 92$, is represented precisely once. Stamp in (6.8) right panel from Japan "Happy" Champagne bottle and glasses: Mi:JP 8882 (22 November 2017).

The parent matrix $\mathbf{M}$ in displays (6.5)–(6.7) above has index 1 and is EP. We may, however, choose, $\mathbf{M}$ (and hence its binary partner $\mathbf{B}$) to be any one of the 48 classic most-perfect pandiagonal magic matrices. Precisely 8 of these 48 have index 1 and are EP and precisely 8 of these 48 have index 3. The other 32 have index 1 but are not EP.
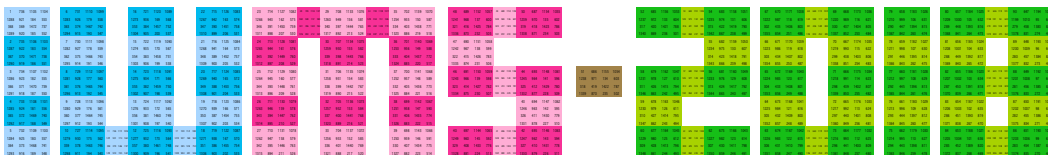
Reference:
[1] ⋆Happy 2018⋆ Equal Sums Magic Squares of Order 4, published by Inder J. Taneja, personal communication from Inder J. Taneja to George P. H. Styan (29 December 2017), and on-line at the *Numbers Magic: Magic Squares, Selfie Numbers and Information Measures* website:

https://inderjtaneja.com/2017/12/29/2018
-equal-sums-magic-squares-of-order-4/

(6.9)

6.6. Talk M3c/Poster Q3: Selected new mathematical postage stamps from USA, Italy, Croatia.
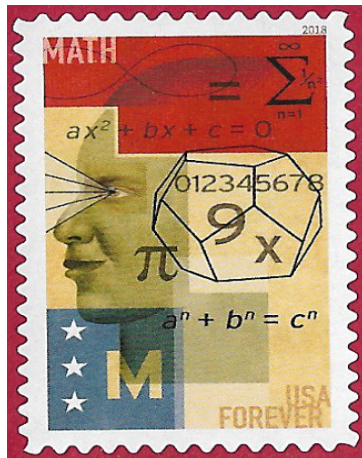
The United States Postal Service (USPS) issued a set of four nondenominated forever stamps on 6 April 2018 to commemorate the subject of STEM Education [1]. Each stamp characterizes one of the four educational disciplines represented by the STEM acronym: Science, Technology, Engineering and Mathematics.

The STEM Mathematics M-stamp shows a lemniscate (top left) and the Basel Problem (top right) which we observe has solution

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

as established by James Gregory FRS (1638–1675) and Leonhard Euler (1707–1783).

The Italian mathematician and philosopher, Maria Gaëtana Agnesi (1718–1799), was the first woman to write a mathematics handbook and the first woman appointed as a mathematics professor at a university. Her "witch of Agnesi" cubic curve (1748) is equivalent to the probability density function of the Cauchy distribution [2] and approximates the spectral energy distribution of spectral lines, particularly X-ray lines.

Marin Getaldić/Marino Ghetaldi (1568–1626) was born in Ragusa, Dalmatia (now Dubrovnik, Croatia) and was a mathematician and physicist who specialized in optics. He was one of the few students of Franciscus Vieta = François Viète, Seigneur de la Bigotière (1540–1603), a French mathematician who was a lawyer by trade, and who served as a privy councillor to both Henri III and Henri IV of France.

References:
[1] "USPS offers STEM Education commemorative stamps; April 6 event at D.C. science and engineering festival", by Michael Baadke, *Linn's Stamp News* online (15 March 2018).

[2] "Cauchy and the witch of Agnesi: an historical note on the Cauchy distribution", by Stephen M. Stigler, *Biometrika,* vol. 61, no. 2, pp. 375–380 (1974).

Updated timetable and programme for the 26th International Workshop on Matrices and Statistics (IWMS-2018), edited & prepared by:

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7. Updated timetable: Tuesday, 5 June 2018

11:00 am/11h00: Registration with pre-lunch coffee break in room 3F.43 and lunch break

2:00 pm/14h00: Mini-symposium celebrating the 100th birth anniversary of Theodore Wilbur Anderson, Jr. (1918–2016) today on 5 June 1918, in rooms 3F.37, 3F.38, 3F.43

- "Some comments on T. W. Anderson's publications", by George P. H. Styan
- "An annotated and illustrated bibliography with hyperlinks for T. W. Anderson in celebration of his 90th birthday", by Simo Puntanen & George P. H. Styan

8. Updated timetable: Wednesday, 6 June 2018

IWMS-2018 invited/contributed talks, rooms 3F.37, 3F.38

- 10:00 am/10h00: Oskar Maria Baksalary & Götz Trenkler: Representations of the Moore–Penrose inverses of matrices
- 10:30 am/10h30: Adi Ben-Israel: Matrix volume and its applications
- 11:00 am/11h00: *Coffee break:* room 3F.43
- 11:30 am/11h30: Dennis S. Bernstein: Input estimation for linear discrete-time dynamical systems
- 12:00 noon/12h00: S. W. Drury: Some old conjectures of Bapat and Sunder
- 12:30 pm/12h30: Yonghui Liu: Estimation and influence diagnostics for an autoregressive model under skew-normal distributions

- 1:00 pm/13h00: *Lunch break*
- 2:30 pm/14h30: Mika Mattila: The arithmetic Jacobian matrix and determinant
- 3:00 pm/15h00: Christopher C. Paige: The effects of loss of orthogonality on large sparse matrix computations
- 3:30 pm/15h30: *Coffee break:* room 3F.43
- 4:00 pm/16h00: Yongge Tian: Multilevel statistical models, least-squares estimators, and reverse-order laws for generalized inverses
- 4:30 pm/16h30: Jorge Delgado Gracia, Guillermo Peña & Juan Manuel Peña: Schoenmakers–Coffey matrices
- 5:00 pm/17h00: Simo Puntanen & Augustyn Markiewicz: Linear prediction sufficiency in the misspecified linear model
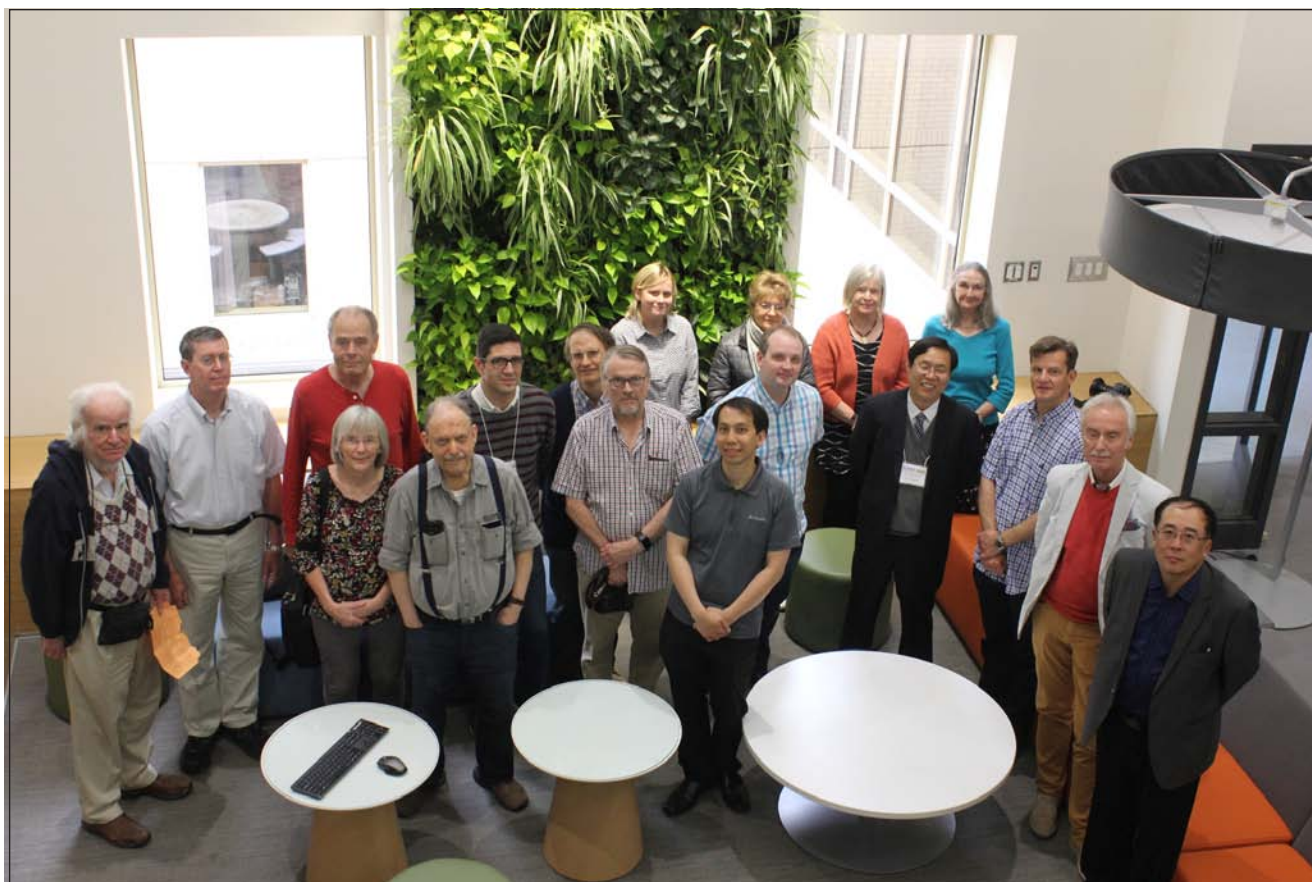
9. Updated timetable: Thursday, 7 June 2018

Mini-symposium on "Magic squares, prime numbers and postage stamps" organized by Ka Lok Chu, Simo Puntanen & George P. H. Styan, in rooms 3F.37, 3F.38, 3F.43

- 10:00 am/10h00: Some comments on a philatelic magic square for the Trenkler–Trenkler MPPD prime-magic matrix
- 10:30 am/10h30: Some comments on two logos and on some related curiosity- and philatelic-friendly numbers
- 11:00 am/11h00: Some comments on selected new mathematical postage stamps from USA, Italy and Croatia

10. Group Photos

Group Photos at IWMS-2018[3]