Effect of adding regressors on the equality of the BLUEs under two linear models

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\textbf{ARTICLE INFO}

\textbf{Article history:}
Received 7 July 2008
Received in revised form 19 June 2009
Accepted 19 June 2009
Available online 27 June 2009

\textbf{MSC:}
15A42
62J05
62H12
62H20

\textbf{Keywords:}
Best linear unbiased estimator
BLUE
Frisch–Waugh–Lovell theorem
Gauss–Markov model
OLSE
Ordinary least squares
Orthogonal projector
Partitioned linear model
Reduced linear model
Updating linear regression

\textbf{ABSTRACT}

In this paper we consider the estimation of regression coefficients in two partitioned linear models, shortly denoted as $\mathcal{M}_{12} = \{y, X_1\beta_1 + X_2\beta_2, V\}$, and $\mathcal{M}_{12} = \{y, X_1\beta_1 + X_2\beta_2, V\}$, which differ only in their covariance matrices. We call $\mathcal{M}_{12}$ and $\mathcal{M}_{12}$ full models, and correspondingly, $\mathcal{M}_i$ and $\mathcal{M}_i$ small models. We give a necessary and sufficient condition for the equality between the best linear unbiased estimators (BLUEs) of $X_1\beta_1$ under $\mathcal{M}_{12}$ and $\mathcal{M}_{12}$. In particular, we consider the equality of the BLUEs under the full models assuming that they are equal under the small models.

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\textbf{1. Introduction}

In this paper we consider the partitioned linear model

\[ y = X_1\beta_1 + X_2\beta_2 + \epsilon, \]  

or more succinctly,

\[ \mathcal{M}_{12} = \{y, X\beta, V\} = \{y, X_1\beta_1 + X_2\beta_2, V\}, \]  

where $E(y) = X\beta$, $E(\epsilon) = 0$, $\text{cov}(y) = \text{cov}(\epsilon) = V$. We denote the expectation vector and covariance matrix, respectively, by $E(\cdot)$ and $\text{cov}(\cdot)$.